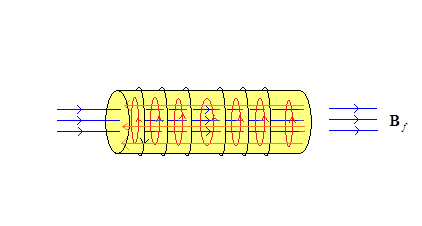
**Energy stored in the Fields**

**Energy stored in Magnetic field within Insulator**

Now let’s calculate the work required to set up a free current distribution **j**f (which produces the blue **B**f field lines below). So what would happen is, we use some force to accelerate the free charges and create a current **j**f. In the process, **B**f. will change from 0 to some final value, and as a result of this change in magnetic flux, a couple things will happen. First, an electric field **E**f field will be induced, which will work against the free current, and also drive bound currents **j**b (in some direction), creating another magnetic field **B**b. And as **B**b itself rises from 0 to its final steady state value, it will create an induced electric field **E**b (temporarily) which will work against the bound currents, but push the free currents (assuming the bound currents are going opposite the free currents).



So the work done we’re doing on the free charges as they are being accelerated, during a particular time interval δt is (ρE = force density, and our force is the same as electric force because we’re looking for minimum work).



Assuming we’re doing it very slowly so that the electric field isn’t changing with time. (we want the minimum necessary energy and so we do things slowly), then we can say that



Filling this in…



Integrating by parts we have,



And inserting another result from Maxwell’s equations:



we have,



and,



Now let’s assume we have a linear medium. Then we can say **H** = **B**/μ, so that we can say,



Integrating from ti to tf, we can conclude



Again, this is the work required to set up the free currents.

**Work required to magnetize an insulator**

So this is a useful quantity to work out, especially with regard to Thermodynamics. So this work should just be the work the electric field generated by the free charge does on the bound molecular charges. And this is:



Leaving off the implicit b subscript, and considering the work done during a small time interval, we can just say:



I’ve seen another formula for this: δW = Bf·dM. But I don’t think it’s true. I think it’s just a heuristic to classically justify the form of the Hamiltonian for a spin in a magnetic field. Also, if our substance has the requisite geometry, like solenoidal, say, and if it’s also linear, homogeneous, etc., then we could equate Bf = μ0H.

**Other ways to write the Energy**

There are other useful ways to write this. Go back to:



Now let’s insert the relation **H** = **B**/μ0 – **M**. So,



which is:



So we can interpret this as the work required to set up the bulk interstitial magnetic field, plus (- sign is part of this) the work the field does on the magnetic dipoles. If we compare this to the formula we get when we apply the 1st Law of Thermodynamics, we’ll see that the last term in the box must be equal to the total change in dipole energy (+ heat exhausted, in the paramagnetic/ferromagnetic case). Effectively we are supplying energy for both of these things. Another way to write it is:



*If* we have a linear medium, so that H and M are proportional to one another, we can write this as:



This is basically the energy stored in the free field, minus the work the free field does on the magnetic dipoles. Now let’s broaden our perspective by placing our analysis in the context of the 1st Law of Thermodynamics. First I’ll consider my classical diamagnet model, generally. Then I’ll consider paramagnet / ferromagnet stuff. So in the former case, the work we do on the system should be equal to the change in energy of the system, according to the first law, presuming no heat transfer. So we should have:



Like with the electric field scenario previously, I believe we can say:



still holds I believe. The meaning of the PE terms is basically thus. Keeping the bound current nil, we may start up the free current, which will require work against the back emf its magnetic field generates (PEff). Then we can start up the bound current, which will require work against the back emf its magnetic field generates (PEbb), and we would also at the same time require work done on the free current (PEfb), keeping it steady against the back emf generated by the bound current magnet field again. So this is the physical origin of the three potential energy terms. And the currents would then run at their final values indefinitely (battery is only necessary to build them up, not to perpetuate them). If a resistance is introduced into the wires, then the resistor will slow the currents down, but the PE aforementioned will then be released into the charges, trying to speed them up. Of course, some of this PE will be dissipated by the internal mechanisms of the bound currents, just as Wf was when building them up. Only if the process were completely reversible, would all of Wf go back into KEf. Anyway, I’d like to explore the total PE again. We can perform the same intricate analysis we did in the E field file, abetted by the considerations in the EB Energy Thoughts file.



Going to have to be careful with the bb terms again because the continuum approximation we’ll do in a bit won’t pick up the **B**bi·**B**bi terms, as our continuum approximation is a dipole approximation, which only works outside the dipole. So we’ll separate out those terms.



Now when we go to the continuum approximation, the second sum in the red term can be set to **B**b since the continuum approximation will ignore ‘self-energy’ terms like **B**bi·**B**bi anyway. So we can say,



Now can combine the first three terms to get:



So basically, like before we get the contribution from the bulk-interstitial field, and the individual dipole-dipole contribution, which must be carefully separated. So now we can say:



Note that sometimes the last *two* terms are combined into one, and written as: (1/2μ0)∫dτ B2. This is true, but in that case, the text formula B would be refering to *both* the bulk/interstitial B (from which we get the first of the last two terms above in the equation) and the individual dipolar B’s (from which we get the last of the two terms above). This process is identical to what we explained in the previous Electric field energy file. Anyway, we’re going to rather combine the two exclusively bound current terms into one:



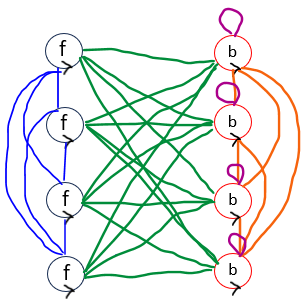
and then we could say:



Now we’ll presume that the force we’re exerting on the free charges, to accelerate them into a current, is actually equal and opposite to the induced electric field force oposing their acceleration. *So technically ΔKEf = 0*. But, at least in our diamagnetic example, ΔKEdipoles won’t be zero – this is the kinetic energy manifested in the induced dipole currents that create the bound current field. And so:



And ΔU = ΔUsys is the work W that is calculated up above in the previous sections – the energy stored in the diamagnet. So we see that the work we do is just the change in energy of our system = change in KE of our induced dipole currents and the change in PE (coming from each dipole’s *own* self-inductance) of starting up each dipole current + bulk change in PE of our dipoles and free currents. This latter term is the work required to start up all the free and bound currents, accounting for the interaction of each bound current with all the other currents (free or bound) and of each free current with all the other currents (free or bound), except for the work term accounting for the resistance of each current from itself. The work term stemming from the restistance of each current with itself is what the first term calculates. One more time, I have a drawing below. The blue terms are PEff green terms are PEfb, and orange terms are PEbb. This is what (1/2μ0)∫B2dτ takes care of. Note there is no self-inductance loopy things in PEff and that’s because the bulk interstitial magnetic field in 3D is diffuse enough that no self-energy terms dipolely terms should exist I don’t think (kind of like how the electric field PEff term shouldn’t have any self-energy terms). The purple terms are KEb + PEb, and this what ΔUdipoles takes care of.



But what about our paramagnet/ferromagnet? As discussed in the model dimagnet file, we cannot construct a paramagnet w/o quantum mechanics and thermodynamics/statistical mechnanics. So that requires some changes. First, we have to allow for heat transfer, since only at constant T will the susceptibility be constant, and since increasing the field does work on the paramagnet, which increases its temperature, we need to allow it to release heat, to keep it down. Furthermore, we know from the equation of state S = S0 – (1/2α)M2 for instance, that increasing the field adiabatically will not change the magnetization. So we’ll have:



where Q is the heat aded (- heat released). And another change is that we’d replace the kinetic energy of our paramagnetic with KEb = -**M**·**B**(ΔV) at least according to our simplest model of a Paramagnet [see QM/Many Particles/Zeeman]. So defining,



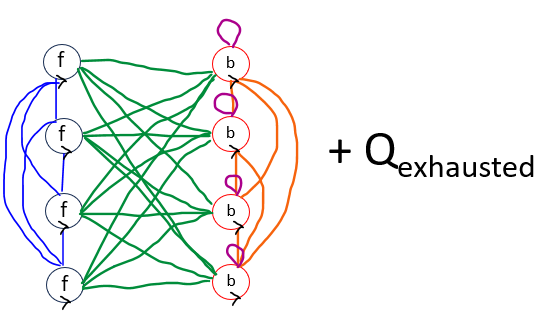
we have:



But then like before, we’ll be doing the minimum work so that KEf = 0, so:



but where again we’re claiming W = ΔU = ΔUsys – Q is the energy stored via work. So we see that the work we do is just the change in energy of our system + heat exhausted = change in KE of our induced dipoles + change in potential energy of our dipoles and free currents + heat exhausted to environment (Q = heat added, so -Q = heat exhausted). Can represent the work done as a diagram again,

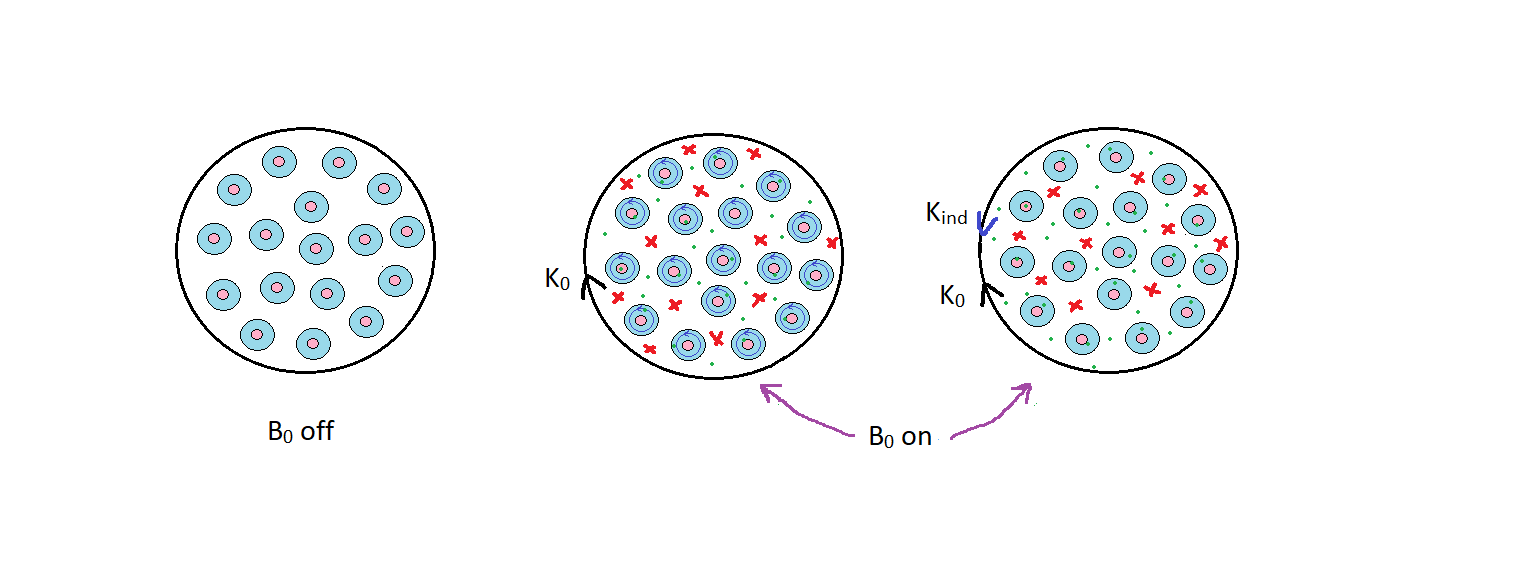


This time the purple bound self-energy term just comprises KE. Might note that as we ramp up **B**, KEb will diminish. This is consistent with the fact that KEb = -**M**∙**B**ΔV as we know the dipoles will align more with the field. Evidentally it diminishes more than Qexhausted is positive because since μ > μ0, we have B2/2μ < B2/2μ0 in this paramagnetic/ferromagnetic situation.

We’ll also observe that KEb = -**M**∙**B**ΔV would *increase* for a *diamagnet* since it would tend to align anti-parallel to the field, and this is consistent with what we found for our classical calculation, in which case KEb is literally the classical kinetic energy of the dipole currents. In all cases, the only dipole energy that we *actually* have is the quantum mechanical energy term KEb = -**M**∙**B**ΔV. The classical diamagnetic scenario I use as a model is just a convenient fiction. And that classical PEb wouldn’t *really* exist. So in all cases, I guess we can treat ΔUdipole as *really* just being the change in quantum mechanical KEb. And in both the diamagnet and paramagnet/ferromagnet cases, there would probably be a concomittant Qexhausted as well. So I guess, succinctly, we could say that for diamagnets, (1/2μ)B2 > (1/2μ0)B2 b/c we’re adding the increase in KEb of the dipoles (+ Qexhausted). And for paramagnets/ferromagnets, (1/2μ)B2 < (1/2μ0)B2 b/c we’re adding the *decrease* in KEb of the dipoles (+Qexhausted but I guess it’s not enough to compensate for decrease in KEb).

**Model Dimagnetic Energy Calculations**

So reconsider a classical diamagnetic placed in between a solenoid. This is a top-down view. The solenoid is the outer cylinder (infinite length, technically) And I’ll model the atoms in the material as tiny radius R cylinders (also infinite length) with a positive cylindrical nucleic charge q, centered about a cylindrical electronic cloud of charge -q. I’ll take these atoms to be a sort of metallic entity so that the charge cloud can sort of flow around the nucleus w/o resistance. And we can ultimately think of the multitude of tiny nuclear currents as constituting one ‘big’ surface current around the perimeter.



and we found, in that file, that:



and also,



which correlated to an effective bound (induced) surface current circumambulating the perimeter:



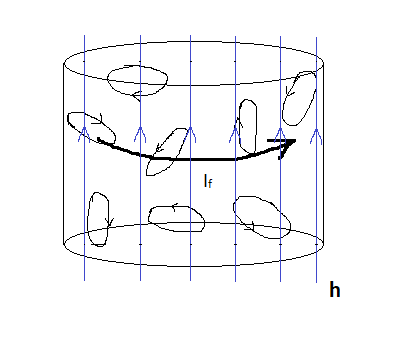
And so the bound field is given by:



And we found therefore that the total field was:



Can also use these expressions to relate the bound current to the free current, Bb = χ­m/(1+χm)B = χm/(1+χm)∙κmB0 = χmB0. And so Kb = χmK0. Then apropos our paramagnet calculation, had the model:



But we had to acquiesce to a quantum mechanical and thermodynamical model, which amounted to, in the low B, high T limit [see stat mech file/quantum paramagnet and field]:



where B is field and T is temperature. And magnetization (of dipoles) given by:



which, parenthetically, enables us to write:



Really all of these formulas would be *per unit volume* or something. Observe how at constant S, we have constant M, and how if B is increasing, we must have that T is increasing.

**Energy as work required to assemble the free currents (as dimagnet slowly magnetizes)**

So the energy stored in our dimagnetic is the work we do on the system comprising the free and bound charges. The work we do is just on the free currents (which themselves do work on bound currents, but that doesn’t matter) And what is the work required to build this up, i.e. what is Wf? We already ‘worked’ this out above, in general. But in the context of our model, this is [note I0 is synonymous with If], quite generically:



[note all ΔV = volume of sample, ΔVcloud = volume of cloud/molecule, and ΔVinduced = potential difference]

**Energy as bulk/interstitial magnetic field potential energy + work it does on bound charge/dipoles**

The first boxed ‘alternate’ energy formula suggested the interpretation that the energy is just the bulk/interstitial PE + the work the bulk/interstitial field does on the bound charges. So we can say, quite generally:



So, because M and B are linearly related, this reduces to:



So there we go.

**Energy as free field potential energy minus work it does on bound charge/dipoles**

The second boxed formula in the ‘alternate’ energy section suggest the interpretation that the energy is just the free field PE – the work it does on the bound charge things to magnetize them. This assumed a linear medium. I’m going to get there a different way, using some stuff in that Energy Musing file that also presume a linear medium.



(did this for classical diamagnet, but with small changes in reasoning, the result applies to the para/ferromagnet too. So continuing,



But again, this doesn’t really make sense to me. I don’t see why on physical grounds the total energy stored ought to be the free current potential energy, minus the work it does on the bound charges. Seems we could say it would be the free current potential energy + the work required to push the dimagnetic into the solenoid + work required to maintain the free current at its value as we push the solenoid in (have to do work against the emf generated by the changing magnetic flux induced by dimagnet). So -Wf|b = Wus|dimagnet + Wus|f.

**Energy as change in internal energy of system (classical diamagnet)**

So for our classical diamagnet, we alleged that the energy stored W = ΔU is just the change in energy of our diamagnet, including the energy of the field, ΔUsys. And this was:



where



Now PEb = NPE1 where N is number of our cylinders, and PE1 is just the change in interaction potential energy between molecular current and its own induced field for one of these cylinders. This is the thing which is not captured by our dipole approximation to the field. So what is PE1? It is the work, W1, we have to do against the molecule’s own induced field to set up its current. And this is:



Okay and now,



(the penultimate step presumed ΔV = NΔVcloud, but we didn’t have to make such an assumption in the electric case. hmmmm) Well now,



so,



And now for the KEb. This would be:



(note that given the range over which our χm can run, this term is in fact positive) Altogether for ΔUdipole, we have:



Putting these into our formula, we have:



So there we go!

**Energy as change in internal energy of system minus heat exhausted (paramagnet)**

So for our paramagnet we said the energy stored, W = ΔU, in the paramagnet was just the work done on it, which was the internal energy it acquired, minus the heat added (i.e. + heat exhausted), i.e. ΔUsys - Q. So,



where,



And for our simple paramagnet, we said (see Stat Mech/Quantum Paramagnet + Field) internal energy, entropy, and magnetization are given by:



Now for dipoles with constant moment, their individual fields do not change magnitude – just orientation. So the ΔPEb term ought to be zero. That gives us:



And then apropos the heat exhausted, we can use the 2nd Law of Thermodynamics to conclude Q = TΔS (at least for quasi-equilibrium process and constant T):



Putting these together we have:



So it checks out 😊.